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AN ANALYSIS OF SNOW AND WIND LOADS COMBINATIONS BASED ON METEOROLOGICAL DATA

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The objective of this paper is to present a probabilistic method of analyzing the combinations of snow and wind loads using meteorological data and to determine their combination factors. Calculations are based on data measured at twelve Polish meteorological stations operated by the Institute for Meteorology and Water Management. Data provided are from the years 1966 - 2010. Five combinations of snow load and 10-minute mean wind velocity pressure have been considered. Gumbel probability distribution has been used to fit the empirical distributions of the data. As a result, the interdependence between wind velocity pressure and snow load on the ground for a return period of 50 years has been provided, and the values of the combination factors for snow loads and wind actions are proposed.

Keywords: snow load, wind velocity pressure, probability distribution, combination of actions, combination factor

1. INTRODUCTION

Various combinations of actions have to be taken into consideration in structural design calculations, and they usually contain two climatic actions: snow and wind loads. According to the current codes of practice it is assumed that characteristic values of each action separately may be exceeded in a particular year with the probability of 0.02 [1] [15 - 17], thus their return period is equal to 50 years. The probability that the characteristic values are exceeded for both actions simultaneously, i.e. snow load and wind load, in a particular year is equal to $0.02^2 = 0.0004$, and the

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return period of such an event is 2.500 years. Thus, design calculations are based on combinations of loads where the larger variable load is not reduced and the remaining variable loads are used with combination factors lower than one. Since the fundamental work by Ferry Borghes and Castahneta [3], combinations of loads have been discussed in numerous publications, e.g. [2] [5 - 13] [18 - 25]. A special issue of Structural Safety was devoted to the combinations of actions, e.g. with papers [7] [11] [13] [25]. Some of the mentioned publications have been used in the work reported in [17] to determine the values defined in the Eurocode [14].

The values of the combination factors (ψ_0 factors) have usually been determined based on theoretical considerations, such as those mentioned above. However, as climatic actions depend on the conditions present in different countries, it is necessary to analyze the available historical meteorological data. The objective of these analyses should be to verify the standard combination factors of variable climatic loads and to identify the values determined on the basis of local data. This is the subject of the article. The initial work was done many years ago, but only on the basis of data from one meteorological station and 20 years of observations [18 - 19]. Recently, it has been possible to perform calculations based on the data measured at twelve meteorological stations operated by the Institute for Meteorology and Water Management - a State Research Institute located in different snow load zones throughout Poland (Fig. 1, Table 1) shown below.

Fig. 1. Meteorological stations whose data were used in the analysis.

2. ANALYZED COMBINATIONS OF SNOW AND WIND LOADS

When analyzing climatic actions, the annual observation period should be assumed as equal to the climatic year starting on October 1st and ending on September 30th. With this assumption, the maximum winter snow load on the ground is also the maximum annual value considered. Maximum wind speeds, 10-minute mean values, subject to probabilistic analysis to define a design value, occur in Poland (and other Central European countries) from autumn to spring. According to recommendations given by the Eurocode [14] these have been used to determine the characteristic values of wind velocity in the Polish National Annex. Therefore, in the case of wind actions, the assumption that the annual observation period starts on the first of October and ends on the thirtieth of September is justified as well. Such givens are much better than analyzing annual wind maxima using calendar years. In the case of snow loads it would be simply impossible, as every winter would need to be divided into two sections.

Table 1. Meteorological stations whose data were used in the analysis.

Meteorological station	φ_N	λ_E	A, m a.s.l.
Chojnice	53^0 42’	17^0 33’	172
Katowice	50^0 14’	19^0 02’	284
Kętrzyn	54^0 05’	21^0 22’	108
Łódź	51^0 44’	19^0 24’	187
Poznań	52^0 25’	16^0 50’	86
Rzeszów	50^0 06’	22^0 03’	200
Siedlce	52^0 11’	22^0 15’	152
Szczecin	53^0 24’	14^0 37’	1
Tarnów	50^0 02’	20^0 59’	209
Warszawa Bielany	52^0 17’	20^0 58’	98
Włodawa	51^0 33’	23^0 33’	175
Wrocław	51^0 06’	16^0 53’	120

Five combinations of snow and wind actions have been considered [28]. The maximum annual values are of key importance: the snow load on the ground and the 10-minute mean wind speed at anemometer height, regardless of the wind direction and the type of the terrain.

Combination 1: maximum annual snow load and wind speed values, measured in the same climatic year but not at the same time. The forecast values may be exceeded once during the T time, in the same climatic year but not on the same day. This is an extreme case, which constitutes the upper limit of all possible combinations.

Combination 2a: maximum annual snow load and the 10-minute mean, maximum daily wind speed value (selected from the values recorded every hour), measured on the same day as the snow load.

Combination 2b: maximum annual snow load and the 10-minute mean, maximum daily wind speed value measured over a 15-day period, with the day on which the maximum annual snow load was measured falling in the middle of the period. It was assumed, quite arbitrarily but based on earlier analyses [26 - 27], that the snow cover whose weight is not much lower than the maximum annual measured value, may remain for two weeks; one week before and one week after the maximum value was measured. A similar approach can be found in the paper of Wang and Rosovsky [22]. As it was shown [26 - 27], maximal values of winter snow loads usually persist over a period of two, three, or four days. Moreover, snow load increases rather slowly, yet decreases faster. For this reason the aforementioned assumption may be considered as a conservative one.

Combination 3: the 10-minute mean, annual maximum wind speed, and snow load measured on the same day. In this combination the snow load was often missing as there was no snow cover; in the analysis the snow load was assumed to be $S = 0$ kN/m^2.

Combination 4: the 10-minute mean, annual maximum wind speed, and snow load measured on the same day, provided that there was a snow cover present. In this combination snow loads were analyzed only if $S > 0$.

3. ASSESSMENT OF THE CORRELATION BETWEEN ANALYZED DATA

An important element of the analysis of the data is evaluation of the correlation, i.e. determining whether the analyzed values are independent or mutually correlated. An evaluation of the correlation between snow load and wind speed is shown in Figures 2 and 3, using the examples of data from the meteorological stations in Kętrzyn and in Rzeszów Jasionka. These figures present data selected according to aforementioned combinations 1, 2a, 2b and 3. The determination coefficients, corresponding to linear regression, are the following: Fig. 2a, combination 1: R^2 = 0.0149; Fig. 2b, combination 2a: R^2 = 0.0004; Fig. 2c, combination 2b: R^2 = 0.0105; Fig. 3, combination 3: R^2 = 0.0009. Such low values of the coefficient of determination may be considered as a lack of correlation. A similar situation occurs in the case of data measured at other meteorological stations.

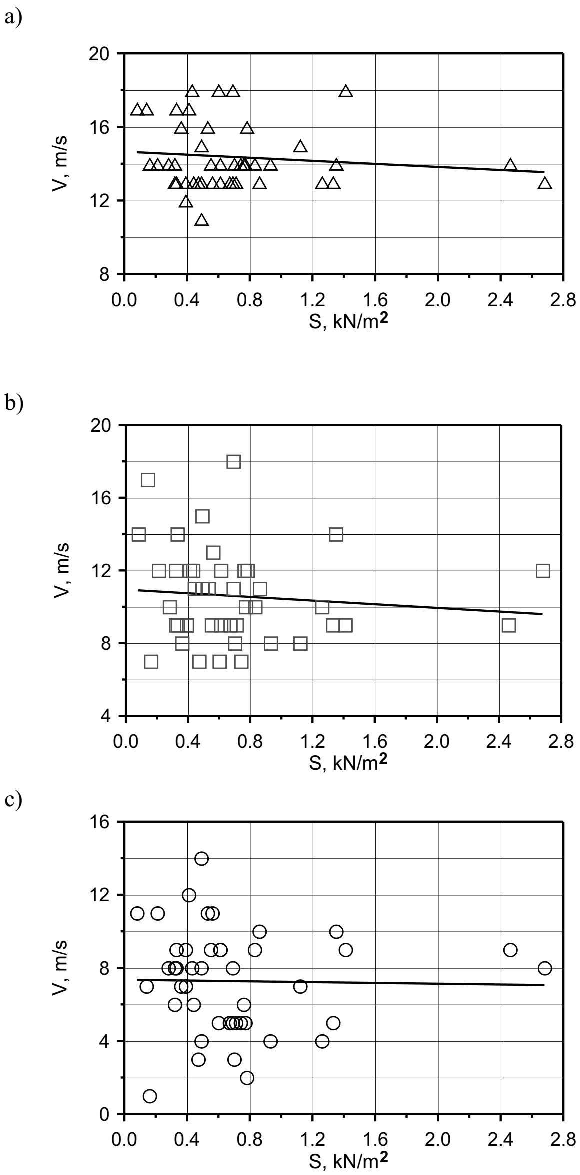


Fig. 2. 10-minute mean wind speeds versus maximum annual snow loads on the ground in three combinations: a) combination 1; b) combination 2a; c) combination 2b. Meteorological station Kętrzyn.

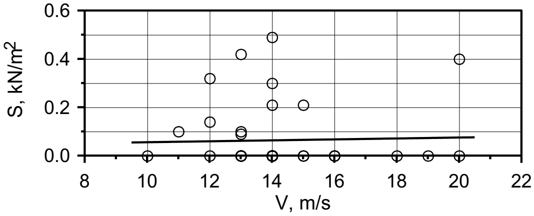


Fig. 3. Snow load on the ground measured in combination with the 10-minute mean annual maximum wind speed (combination 3). Some values of *V* and the simultaneously measured values of $S = 0$ were recorded either several or over 10 times (see Fig. 5). Meteorological station Rzeszów Jasionka.

Consequently, it may be assumed that in all the analyzed combinations the two random variables (the snow load on the ground and the wind speed) are not correlated.

4. METHOD OF ANALYSIS

The task set consists of finding values of snow load on the ground S_T and wind speed V_T which are simultaneously exceeded once in *T* years.

If it is assumed that in the observation period both snow cover and wind are present and the probability density functions for snow load $f_1(S)$ and wind speed $f_2(V)$ are known, then the probability that the wind speed *V* is within the range of $V_1 < V \le V_2$, when the snow load on the ground is in the range of $S_1 < S \le S_2$ is equal to [18-19]:

$$\Phi[V_1 < V \le V_2; S_1 < S \le S_2] = \int_{S_1}^{S_2}\int_{V_1}^{V_2} f_1(S)\cdot f_2(V)\cdot dS\cdot dV\,. \tag{4.1}$$

From the standpoint of the safety of a structure, it is important to determine such combinations of the snow load and the wind load which will be exceeded, with an acceptable probability in the assumed reference period. Usually, it is assumed that the probability of exceeding the characteristic value in the reference period of one year is equal to 0.02; thus, the frequency of such an event is once every 50 years. Other assumptions can of course be made, if they are accepted by appropriate authorities.

If $P(S_T, V_T)$ is the probability of an event where the S_T and V_T values are exceeded simultaneously in the T period, this condition can be written down as

$$P(S > S_T, V > V_T) = \frac{1}{T}. \tag{4.2}$$

If the $f_1(S)$ and $f_2(V)$ functions are integrable and their distribution functions are written as $F_1(S)$ and $F_2(V)$, then the probability of an event where the S_T and V_T values are exceeded simultaneously is

$$P(S > S_T, V > V_T) = P_1(S) \cdot P_2(V) = (1 - F_1(S_T)) \cdot (1 - F_2(V_T)). \tag{4.3}$$

The task is to determine which values of S_T and V_T are exceeded simultaneously in the assumed return period of T years, i.e. to determine the boundary distributions of probability of the snow load on the ground and the wind speed. For this purpose, equation (3) must be solved taking into account the form

$$(1 - F_1(S_T)) \cdot (1 - F_2(V_T)) = \left(\frac{1}{T}\right). \tag{4.4}$$

Assuming that the boundary distribution is the Gumbel distribution [4] and using the distribution function in Eq. (4.4), in the case of snow load according to

$$F_1(S) = \exp(-\exp(-\alpha_s \cdot (S - U_s))) \tag{4.5}$$

and in the case of wind speed according to

$$F_2(V) = \exp(-\exp(-\alpha_v \cdot (V - U_v))) \tag{4.6}$$

the formula obtained is:

$$1 - [\exp(-\exp(-\alpha_v \cdot (V - U_v)))] = \left(\frac{1}{T}\right) : [1 - \exp(-\exp(\alpha_s \cdot (S - U_s)))]. \tag{4.7}$$

This equation must be solved to determine one of the variables in the second function, e.g. the wind speed as a function of the snow load.

Transformation of Eq. (4.7) results as follows

$$V = U_v - \frac{1}{\alpha_v} \ln\left\{ -\ln\left[1 - \left(\frac{1}{T}\right) : (1 - \exp(-\exp(-\alpha_s \cdot (S - U_s)))) \right] \right\}. \tag{4.8}$$

In these Equations, α_s and U_s are parameters of the Gumbel distribution of the maximum annual values of the snow load on the ground, and α_v and U_v are the Gumbel distribution parameters of the maximum annual or daily (depending on the combination in question) 10-minute mean wind speeds. They can be determined using one of the applicable methods, for example the maximum likelihood method.

The reference value of an action comparable to the snow load on the ground is the wind velocity pressure, which is expressed by means of the same units as the snow load. By squaring the right side of Eq. (4.8) and multiplying it by 0.5ρ (where $\rho = 1.25$ kg/m^3) and by converting the value from pascals to kN/m^2, one can determine the interdependence between the wind velocity pressure and the snow load on the ground.

In the analysis of combination 3 a significant difficulty appears: the maximum wind speeds of the annual observation periods, i.e. periods starting on October 1st and ending on September 30th, very often occur when there is no snow cover, in November, for example. This leads to interpretative problems, whose solution may be attempted using combination 4.

In this combination, the frequency of cases where there is no snow cover can be expressed as

$$p_0 = \frac{n}{N} \tag{4.9}$$

where:

N – number of analyzed annual periods,

n – number of analyzed periods with no snow cover at the time of occurrence of the maximum annual wind speed.

Taking into account periods with no snow cover, the probability that the snow load S_p is not exceeded can be calculated using the following equation

$$p(0 \le S_p) = p_0 + (1 - p_0) \cdot \int_{\epsilon > 0}^{S_p} f(x)dx \tag{4.10}$$

where:

S_p – p fractile of the variable of the snow load on the ground,

f(x) – density function of the probability distribution of the snow load on the ground.

In this case, the time series of the snow loads which occur simultaneously with the maximum annual wind speeds comprise N-n values of $S = \epsilon > 0$.

Because

$$F(s \le S_p) = \int_{\in>0}^{S_p} f(x)dx \tag{4.11}$$

Eq. (4.10) can have the following form:

$$F_1(s) = p_0 + (1 - p_0) \cdot F(s). \tag{4.12}$$

Taking into account Eq. (4.4) together with the form of Eq. (4.12), the following function can be formed

$$F_2(V_T) = 1 - [T \cdot \{1 - p_0 - (1 - p_0) \cdot F(s)\}]^{-1}. \tag{4.13}$$

Using the Gumbel distribution to approximate both empirical distributions according to Eq. (4.6), one can present Eq. (4.13) in the following form:

$$F_2(V) = 1 - [T \cdot \{1 - p_0 - (1 - p_0) \cdot \exp(-\exp(-\alpha_S \cdot (S_T - U_T)))\}]^{-1}. \tag{4.14}$$

Instead of Eq. (4.8), one can obtain:

$$V = U_v - \frac{1}{\alpha_v} \ln\left[-\ln\left\{1 - [T \cdot \{1 - p_0 - (1 - p_0) \cdot \exp(-\alpha_S \cdot (S - U_S))\}]^{-1}\right\}\right] \tag{4.15}$$

The parameters of the Gumbel distribution are estimated based on measurement data recorded on days with the snow load value $S > 0$.

The wind speed obtained through Eq. (4.15) can be transformed into velocity pressure, in a way that is similar to the recalculation of Eq. (4.8).

In the case of combination 2a, the three-parameter Weibull distribution can sometimes be more appropriate for the approximation of the empirical distribution of the wind speed; in this case Eq. (4.8) takes another form. As a result of the approximation of the measured data using the latter distribution, one obtains smaller forecast values of the wind speed than when the Gumbel distribution is applied.

5. ANALYSIS OF MEASURED DATA: SCOPE OF CALCULATIONS AND PRESENTATION OF RESULTS

The data obtained from the twelve meteorological stations shown in Fig. 1 and in Table 1 were calculated. The measured values cover a 44-year-long period of observation, from 1 October 1966 until 30 September 2010. The start date was chosen as by the end of 1965 all meteorological stations in Poland were equipped with modern - for that time - wind gauges (anemorhumbometers M-47). The stations were selected in a manner as to cover the entire territory of Poland, mainly along its perimeter with a few more in the center of the country. This way the possible differences in the climatic conditions were taken into account. It may be pointed out that the meteorological stations under consideration are situated in different snow load zones, e.g. Wrocław is in the zone of 0.7 kN/m^2 and Kętrzyn 1.6 kN/m^2. All stations make hourly measurements of the 10-minute mean wind velocity and daily measurements of the snow cover. However, the water equivalent was measured every 5 days, but also after new snowfall and during the days of towing. Since the 1990s the water equivalent is also measured every day. Thus, it may be assumed that any real annual maxima were not missed.

The calculations were performed according to the aforementioned combinations. The parameters of the Gumbel distribution were estimated using the maximum likelihood method. The wind speeds were calculated irrespective of wind direction. The influence of terrain roughness on the wind speed was not considered either; it can be disregarded as in the final analysis the ratio of velocity pressure values is considered.

Fig. 4 shows an example of the Gumbel distribution used to approximate the wind speeds recorded in combinations 1, 2a, and 2b, while Fig. 5 shows an example of distribution of the snow load on the ground at the meteorological station in Rzeszów on the days of maximum annual wind speeds (combination 4).

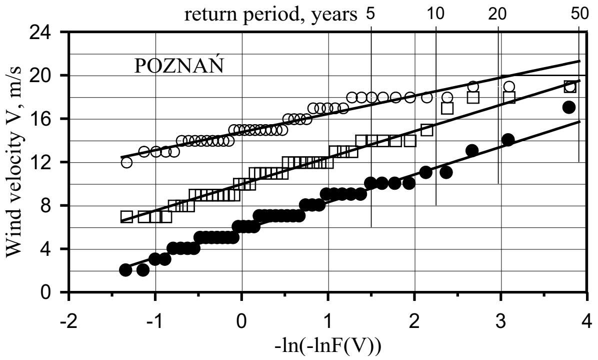


Fig. 4. Empirical distribution of the 10-minute mean wind velocity on the probability plot of the Gumbel distribution. Top set: maximum annual values in combination 1; bottom set: maximum daily values in combination 2a; central set: maximum values across 15 days in combination 2b. Poznań meteorological station.

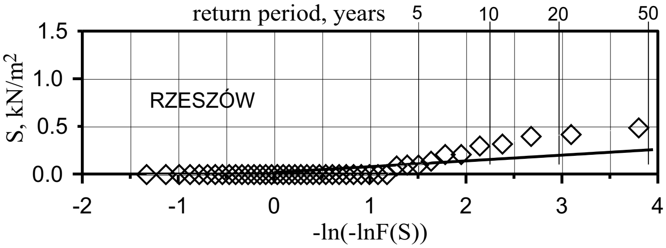


Fig. 5. Boundary empirical distribution of the values of the snow load on the ground measured on the day of occurrence of the maximum annual wind speeds.

This station is one of the few where the snow load was recorded over ten times simultaneously along with the maximum annual wind speed. During the 44 years of observation, at 12 meteorological stations the annual maximum wind speed and snow load are present simultaneously most often 4 times. As the Gumbel distribution is unlimited on both sides, it may formally be used also when the snow load value is zero.

In combination 4 there is a problem with forecasting the snow load based only on several maximum annual values. Both combinations 3 and 4 were considered and the results are presented in the respective Tables and Figures.

The results of the calculations can be presented in the form of three-dimensional drawings, as shown in Fig. 6, in which on the vertical axis there is the probability of exceedance. Such form of presentation was shown in earlier publications [18-19] and can also be seen in other articles, e.g. [22].

The main, target results of the calculations are presented graphically as the interdependence between wind velocity pressure and snow load on the ground for the selected return period. An example of such a graph is shown in Fig.7 for several return periods.

All calculations were performed for the return period of T = 50 years. The examples are shown in Fig. 8.

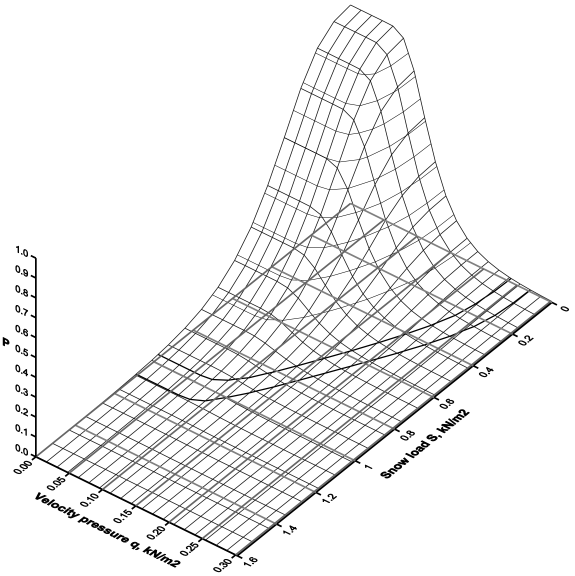


Fig. 6. Interdependence of snow load on the ground, wind velocity pressure and the probability of their simultaneous exceeding. Meteorological station in Rzeszów Jasionka, combination 1.

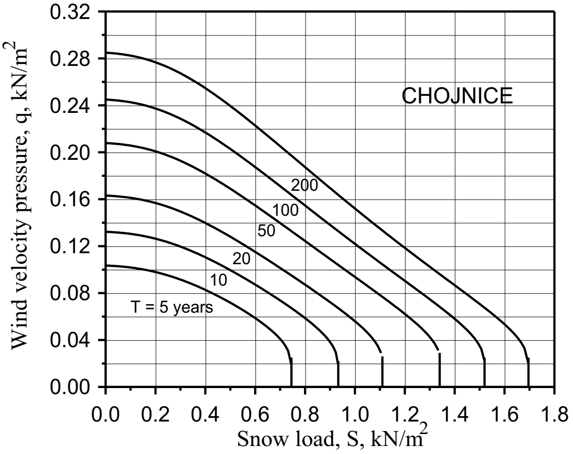
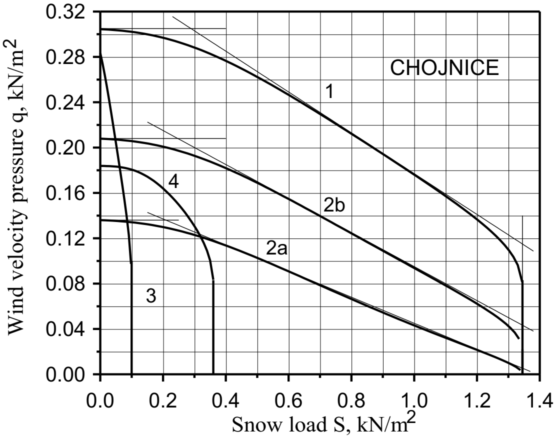


Fig. 7. The interdependence between wind velocity pressure and snow load on the ground for several return periods T. Combination 2b, meteorological station in Chojnice.

In the case of combinations 1, 2a, and 2b, three ranges of interdependence between the two variables can be identified. In combination 1, the first range is the characteristic value of the wind velocity pressure q_k and the corresponding low value of the snow load on the ground. The next section can be replaced with a straight line which is tangent to the curve determined as a result of the analysis in a way that is similar to that shown in the work on structural safety by Mathieu [8]. The third section is the characteristic value of the snow load on the ground S_k and the wind velocity pressure decreasing to zero.

a)



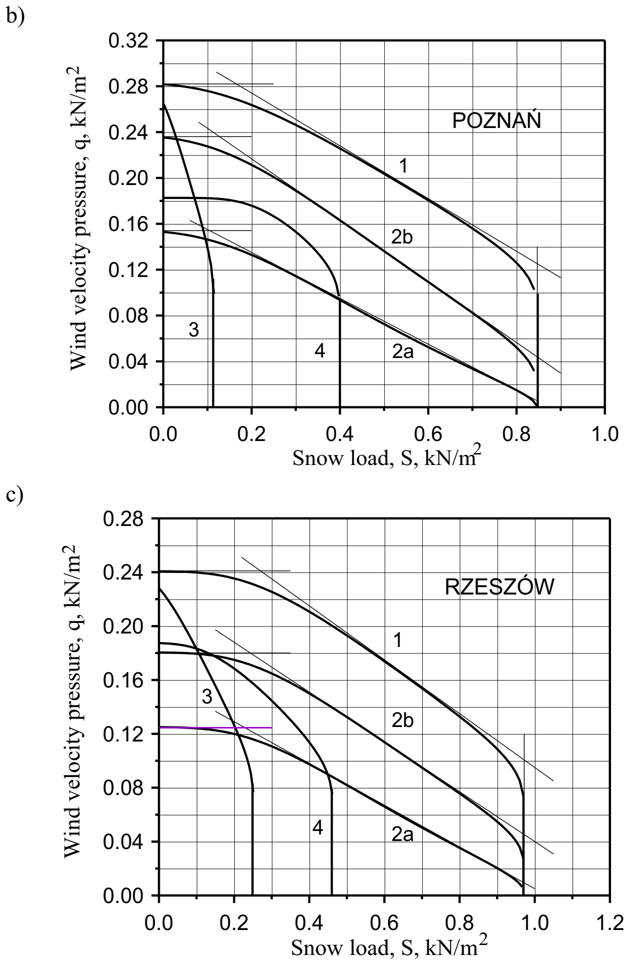


Fig. 8. Examples of interdependence between wind velocity pressure and snow load on the ground which may occur once every 50 years, according to the five combinations. Meteorological stations in Chojnice, Poznań and Rzeszów.

The q(*S*) relationship can thus be shown as three straight lines whose intersections indicate the values which may be included in the load combinations (Fig. 9 and 10). This concept is clarified below.

The first intersection of the straight lines in combination 1 identifies the value of snow load $S(q_k)$ in combination with the characteristic value of wind velocity pressure q_k. In the example given these are: the full value $q_k = 0.24$ kN/m^2 and related value of the snow load $S(q_k) = 0.24$ kN/m^2. The last one constitutes 23.5% of the characteristic value that is $S_k = 1.02$ kN/m^2. The second intersection is the full characteristic value of snow load on the ground and the related value of wind velocity pressure $q = 0.101$ kN/m^2, which constitutes 42.1% of its characteristic value. It is also possible to take into consideration intermediate values, the incomplete characteristic values of both actions, $q < q_k$ and $S < S_k$, according to the tangent straight line which connects both intersections.

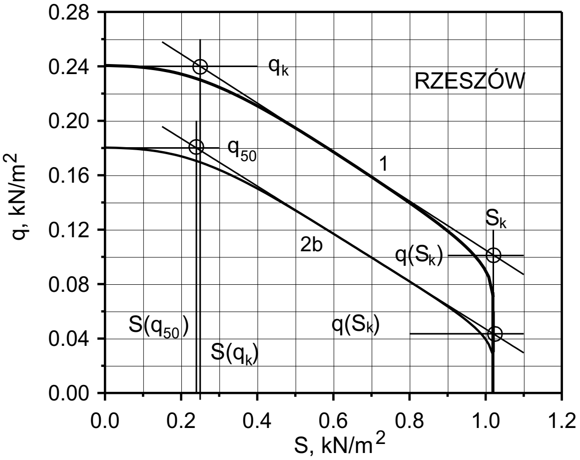


Fig. 9. Instances of interdependence between wind velocity pressure and snow load on the ground for a return period of 50 years. Combinations 1 and 2b. Combination 2b is considered to be most appropriate. Rzeszów Jasionka meteorological station.

The example given pertains to the maximum annual values of wind speeds and snow load occurring in the same observation years, though not at the same time. This is a situation that will not take place in practice. It simply means that once every 50 years the full characteristic value of wind velocity pressure and 23.5% of the characteristic value of snow load, or the full characteristic value of snow load and 42.1% of the characteristic value of wind velocity pressure will occur in the same climatic year but not simultaneously. Curve 1 is a kind of envelope for the possible combinations of snow and wind load. It constitutes a limit beyond which the other curves will not pass.

The characteristic values do not occur in all combinations. For example, in combination 2b the wind speed (and the velocity pressure) which can be exceeded on average once every 50 years does not reach the characteristic value, because it is not the result of an analysis of maximum annual values but of values that are recorded in fifteen-day periods, with the day on which the maximum annual snow load on the ground was observed in the middle. Such values of wind velocity pressure are provided with the 50 index, e.g. q_{50}.

6. FINAL RESULTS OF THE CALCULATIONS

The final forms of the presentation of results are the values of the combination factors ψ_0, for the snow and wind loads. They are presented in Fig. 10 and should be considered in connection with Fig. 9. Factor $\psi_{w,1}$ shows the ratio of velocity pressure q, which corresponds to the characteristic snow load value S_k in combination 1, to the characteristic value of velocity pressure q_k

$$\psi_{w,1} = \frac{q(S_k)}{q_k}. \tag{6.1}$$

Similarly, the $\psi_{s,1}$ factor shows the ratio of snow load S corresponding to the characteristic value of velocity pressure q_k, to the characteristic value of snow load S_k

$$\psi_{s,1} = \frac{S(q_k)}{S_k}. \tag{6.2}$$

Similarly, in the case of combination 2a, the relevant ψ factors are presented as

$$\psi_{w,2a} = \frac{q_{2a}(S_k)}{q_k} \tag{6.3}$$

and

$$\psi_{s,2a} = \frac{S_{2a}(q_{50})}{S_k}. \tag{6.4}$$

The factors for combination 2b are shown in a similar manner. The values obtained are given in Tab. 2.

The final results of the calculations are diagrams of the interdependence between the snow load on the ground and the wind velocity pressure for the return period of 50 years. The curves are illustrated on collective diagrams showing the results of the calculations in the aforementioned five combinations. The examples are depicted in Figure 8. The values read from the intersections of the three straight lines and regarded as combination factors in accordance with Figures 9 and 10 and Eq. (6.1) - (6.4) are given in Table 2. They were calculated based on the data collected at the 12 meteorological stations in three combinations: 1, 2a, and 2b. They are given in relation to the characteristic values of the snow load on the ground and the wind velocity pressure, which are also seen in Table 2.

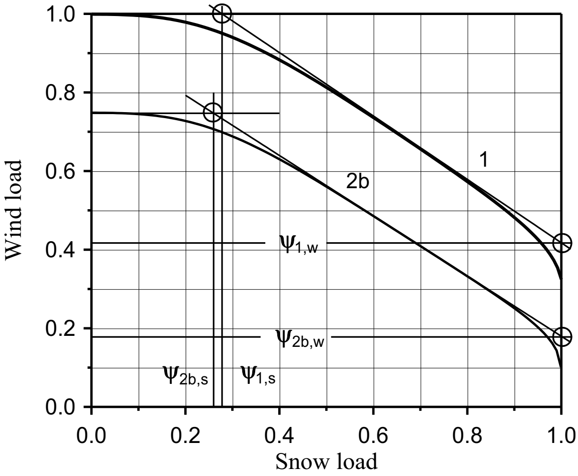


Fig. 10. Designation of factors for combinations of actions (ψ factors).

As mentioned above, the first of the curves, drawn as a result of the analysis of the data for combination 1, constitutes a limit beyond which none of the other curves extends. In the third curve, due to many zero values of snow load on the ground, snow load associated with characteristic values of the wind velocity pressure is very low. For this reason, the maximum value of wind velocity pressure that at $S = 0$ should be the same as that shown in curve 1, is in fact lower, because the value of $q = q_k$ occurs at $S < 0$, which arises from the form of the straight line of the forecast at -ln(-ln(F(S) < 0 (see Fig. 5).

Combination 2b (curve 2b) was found to be most appropriate; it identifies the interdependence between snow load and the associated wind velocity pressure not more than a week before and a week after the maximum value of the snow load. This interdependence can be presented, in a simplified manner, as three straight lines, similar to curves 1 and 2a.

Table 2 also gives the highest and lowest values of the combination factors calculated based on the data measured at the 12 meteorological stations [28], their mean values and standard deviations, as well as the value calculated as the mean value plus three standard deviations, hence with very little probability of being exceeded. It was assumed that the distribution of the values of the combination factors is a normal distribution. Due to the low amount of data, the only approximate confirmation of this assumption is from the mean values of the sum of the maximum and minimum values of the combination factor. They are very close to the mean values of the 12 pieces of data and, in the case of combination 2b, the values are in fact the same. This demonstrates that the distribution of the calculated values is close to a symmetrical distribution in relation to mean values, i.e. close to the normal distribution. Of course, one must keep in mind that there are only 12 values present in each combination.

Given the above, one can suggest that the results of the calculations of both combination factors, i.e. the snow load and the wind load, should be equal to $\psi_{0,s} = \psi_{0,w} = 0.3$. The index used here, i.e. "0", was introduced due to the standard of the symbols used in the Eurocode [14].

The values given correspond to the mean value plus three standard deviations. If the mean values are assumed, then the values would equal $\psi_{0,s} = \psi_{0,w} = 0.2$.

The assumption of the full value of the wind load and $\psi_{0,s} \leq 0.3$ for snow load must be explained. Combination 2b indicates that in such a situation the wind load is equal (on average) to 0.733 of the characteristic value. However, when three standard deviations ($3\sigma = 0.128$) are added to the mean value, then the result is 0.861, which is then rounded up to 1. Consequently, together with $\psi_{0,s} \leq 0.3$ for snow load, the full value of the wind velocity pressure was assumed.

The above-mentioned values were assumed based on the results of the analysis of data in combination 2b, i.e. the data recorded as the maximum annual snow load on the ground and the 10-minute mean wind speed, the highest recorded over a period of 15 days, with the day on which the maximum annual snow load was obtained falling in the middle. However, it must be emphasized that maintaining a value close to the maximum for two weeks is very rare. Such situations may take place in parts of Poland considered as snow load zone 3 or higher, where the average period of snow cover lingering is no shorter than 70 days; in the north-east region and the mountains.

Consequently, the assumption that combination 2b is most appropriate for the purpose of determining the combination coefficients can be considered as conservative.

The results obtained in the calculations according to combination 2a were treated similarly to the results of the calculations according to combination 1; as ones used mostly for the purpose of comparisons. The values of the combination factors are lower than those obtained in the calculations according to combination 2b.

Table 2. Values of the combination factors. Values considered to be most appropriate are emphasized in bold font.

Station	Snow load				Wind load			
	S_k, kN/m^2	$\psi_{0,1}$	$\psi_{0,2a}$	$\psi_{0,2b}$	q_k, kN/m^2	$\psi_{0,1}$	$\psi_{0,2a}$	$\psi_{0,2b}$
Chojnice	1.347	0.218	0.148	0.183	0.305	0.380	0.017	0.141
Katowice	1.072	0.247	0.217	0.238	0.185	0.422	0	0.189
Kętrzyn	1.716	0.233	0.188	0.211	0.224	0.473	0.011	0.176
Łódź	1.149	0.220	0.155	0.202	0.232	0.422	0.021	0.177
Poznań	0.847	0.190	0.124	0.146	0.283	0.442	0.020	0.155
Rzeszów	0.972	0.278	0.233	0.250	0.241	0.419	0.039	0.187
Siedlce	1.307	0.175	0.129	0.135	0.280	0.325	0.016	0.125
Szczecin	0.830	0.202	0.140	0.161	0.204	0.495	0.026	0.216
Tarnów	0.881	0.261	0.207	0.245	0.127	0.732	0.023	0.118
Warszawa	1.081	0.185	0.155	0.168	0.345	0.362	0.015	0.136
Włodawa	1.197	0.231	0.176	0.211	0.208	0.486	0.010	0.236
Wrocław	0.867	0.166	0.115	0.129	0.304	0.319	0.004	0.122
lowest value	-	0.166	0.115	0.129	-	0.319	0	0.118
highest value	-	0.278	0.233	0.250	-	0.732	0.039	0.236
mean value	-	0.217	0.166	**0.190**	-	0.440	0.017	**0.165**
mean standard deviation σ	-	0.035	0.039	0.043	-	0.109	0.010	0.038
variability coefficient	-	0.155	0.232	0.225	-	0.247	0.609	0.232
mean value plus 3σ	-	0.322	0.282	**0.318**	-	0.766	0.048	**0.280**
(min + max)/2	-	0.222	0.174	**0.190**	-	0.523	0.020	**0.177**

An analysis of the results of the calculations according to combination 3 leads to the conclusion that the characteristic value of wind velocity pressure could not be related to snow load. The assumption of full wind load simultaneous with 30% snow load is a conservative proposition as well.

The results obtained according to combination 4, in most cases, are within the limits defined in combination 2b and sometimes go beyond; at low values of snow load, wind velocity pressure values are higher than those in combination 2b. It would be interesting to see the results of the analysis of data measured at meteorological stations located in the mountains, where the duration of snow cover is longer than in other parts of Poland.

7. CONCLUDING REMARKS

This article presents the method and results of an evaluation of combinations of snow and wind loads using the measured values of annual (winter) maximum snow load on the ground and the 10-minute mean annual maximum wind speed; moreover, the article presents the results of the analysis of data collected over a period of 44 years at twelve meteorological stations of the Institute for Meteorology and Water Management - a State Research Institute. The unit observation time was assumed to be the climatic year starting on October 1st.

The interdependence between snow load on the ground and wind velocity pressure for a return period of 50 years was shown. The formula illustrated here can be applied to perform similar calculations for different return periods.

Some comparisons with other provisions may be interesting.

As per the American Standard [1], in the method of load and resistance factor design (LRFD) there are seven combinations of loads from which four may be considered for comparisons. They are as follows:

2. 1,2D + 1,6L + 0,5(L_r or S or R)
3. 1,2D + 1,6(L_r or S or R) + (L or 0,5W)
4. 1,2D + 1,0W + L + 0,5(L_r or S or R)
6. 0,9D + 1,0W,

where: D – dead load, L – live load, L_r – live load on a roof (other than snow or rain), R – rain load, S – snow load, W – wind load.

Numeral coefficients for snow and wind loads (except for the last one) are 0.5. However, this value is a product of safety and combination factors. If the safety factor is assumed to equal 1.4, the combination factor would then equal 0.36, which is not so far from the calculations presented above.

The Eurocode recommends that a combination of actions must be taken into account:

- the design value of the leading variable action and
- the design combination values of the accompanying variable actions.

The combination of actions presented in the Eurocode [14] (containing Eq. 6.10) includes one leading variable action of full design value, and a sum of the design values of the accompanying variable actions, multiplied by the combination factors. This combination is presented as follows

$$\sum_{j\geq l}\gamma_{G,j}G_{k,j}\text{"+"}\gamma_p P\text{"+"}\gamma_{Q,1}Q_{k,1}\text{"+"}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \tag{7.1}$$

where: G – is a permanent action, P – is a prestressing action, if any is present, Q – is a variable action, γ - is a partial factor for relevant action, ψ_0 – is a factor for combination values of a variable action, and "+" implies "to be combined with".

According to the above-presented equation, the design value of the variable leading action is the product of the characteristic value $Q_{k,1}$ and the relevant partial factor $\gamma_{Q,1}$ (partial safety factor). The design value of the accompanying variable action is included in the combination of actions as the product of the characteristic value $Q_{k,i}$, the partial factor $\gamma_{Q,i}$, and the combination coefficient $\psi_{0,i}$.

The following values of the ψ_0 factor are given in the Eurocode [14]: snow load $\psi_0 = 0.50$ (at the altitude of $A \leq 1{,}000$ m above sea level) and wind load $\psi_0 = 0.60$.

The values of the ψ_0 factor proposed here are lower than those given in the Eurocode, namely:

- for snow load $\psi_{0,s} = 0.30$
- for wind load $\psi_{0,w} = 0.30$.

This value, the same for snow and wind actions, may be used when designing structures situated at altitudes not higher than 300 m above sea level, which cover the majority of Polish territory.

An analysis of data from a larger number of meteorological stations would probably enable the assumption that in lowland areas (A ≤ 300 m above sea level) the characteristic value of the wind load is part of the combination without the characteristic value of the snow load. Table 2 demonstrates that the values of the combination factors do not depend on the characteristic values of the snow or wind load, nor do they depend on the zones of both of these loads.

It is intended that the value of $\psi_0 = 0.30$ will be recommended to the Polish National Annex to the Eurocode, i.e. PN-EN 1990:2002 [14]. It is also assumed that the presented method could be used in other Central European countries with similar climatic conditions.

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Tab. 2. Wartości współczynników do kombinacji. Wartości uważane za najwłaściwsze są pogrubione.

ANALIZA KOMBINACJI OBCIĄŻENIA ŚNIEGIEM I WIATREM NA PODSTAWIE DANYCH METEOROLOGICZNYCH

Słowa kluczowe: obciążenie śniegiem, ciśnienie prędkości wiatru, rozkład prawdopodobieństwa, kombinacja oddziaływań, współczynnik do kombinacji.

STRESZCZENIE:

W artykule przedstawiono metodę probabilistycznej analizy kombinacji obciążenia śniegiem i wiatrem na podstawie danych meteorologicznych oraz ustalenia wartości współczynników do kombinacji tych oddziaływań. W obliczeniach wykorzystano dane pomiarowe z 12 polskich stacji meteorologicznych Instytutu Meteorologii i Gospodarki Wodnej – Państwowego Instytutu Badawczego z lat 1966 – 2010. Dane pomiarowe wybrano przy założeniu, że pochodzą one z tego samego roku klimatycznego, trwającego od 1 października do 30 września. Przy takim założeniu wartość maksymalna zimowa obciążenia śniegiem gruntu jest jednocześnie wartością maksymalną roczną. Maksymalne prędkości wiatru, średnie 10.minutowe, występują w Polsce zwykle od jesieni do wiosny; służą one do wyznaczania wartości charakterystycznych. Przyjęcie rocznego przedziału czasu od 1 października do 30 września jako jednostkowego czasu obserwacji jest więc tu również uzasadnione.

Rozpatrzono pięć kombinacji obciążenia śniegiem i średniej 10.minutowej prędkości wiatru, na wysokości anemometru, niezależnie od kierunku wiatru i rodzaju terenu. Są to:

Kombinacja 1: wartości obciążenia śniegiem i prędkości wiatru, maksymalne roczne, zmierzone w tym samym roku klimatycznym, lecz nie jednocześnie. Wartości prognozowane mogą być zatem przewyższone raz w czasie *T*, w tym samym roku klimatycznym, lecz nie w tym samym dniu. Jest to przypadek skrajny, ograniczający od góry wszystkie możliwe kombinacje.

Kombinacja 2a: maksymalne roczne obciążenie śniegiem i maksymalna dobowa prędkość wiatru, średnia 10.minutowa, zmierzona tego samego dnia co obciążenie śniegiem.

Kombinacja 2b: maksymalne roczne obciążenie śniegiem i maksymalna dobowa prędkość wiatru, średnia 10.minutowa, zmierzona w przedziale 15 dni centrowanych wokół dnia, w którym zmierzono maksymalne roczne obciążenie śniegiem. Przyjęto tu, dość arbitralnie, aczkolwiek na podstawie wcześniejszych analiz [19, 20], że maksymalny ciężar pokrywy śnieżnej, lub niewiele od niego mniejszy, może się utrzymywać przez dwa tygodnie, tydzień przed i tydzień po największej wartości zmierzonej. Na podstawie danych z 5 stacji stwierdzono, że w ciągu ok. 10% czasu zalegania pokrywy śnieżnej jej ciężar maksymalny maleje o ok. 5%.

Kombinacja 3: maksymalna roczna prędkość wiatru, średnia 10.minutowa, oraz obciążenie śniegiem zmierzone tego samego dnia. W tej kombinacji często brak obciążenia śniegiem, bo nie ma pokrywy śnieżnej lecz do analizy przyjęto także wartości obciążenia $S = 0$ kN/m^2.

Kombinacja 4: maksymalna roczna prędkość wiatru (średnia 10.minutowa) i obciążenie śniegiem zmierzone tego samego dnia pod warunkiem, że była pokrywa śnieżna. W tej kombinacji do analizy przyjęto tylko przypadki $S > 0$.
Sprawdzono ewentualne korelacje obu zmiennych i stwierdzono, że we wszystkich rozpatrywanych kombinacjach obie zmienne losowe, obciążenie śniegiem gruntu i prędkość wiatru, nie są skorelowane.

Jako brzegowy rozkład prawdopodobieństwa został użyty rozkład Gumbela, którego parametry oszacowana metodą największej wiarygodności. Jako wynik analizy przedstawiono współzależności obciążenia śniegiem i ciśnienia prędkości wiatru, o okresie powrotu obu zmiennych wynoszącym 50 lat. Na tej podstawie zaproponowano wartości współczynnika do kombinacji o wartości 0,3 w przypadku obu oddziaływań. Tę wartość zamierza się zaproponować do załącznika krajowego do normy PN-EN 1990:2002.